GCE Examinations

# Further Pure Mathematics Module FP3

Advanced Subsidiary / Advanced Level

Paper H

Time: 1 hour 30 minutes

*Instructions and Information*

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

*Advice to Candidates*

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

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**1.** Given that

$$t_{n+1} = t_n - 4 \quad \text{for } n \geq 1, \quad t_1 = 3,$$

prove by induction that $t_n = 7 - 4n$ for all integers $n$, $n \geq 1$. **(5 marks)**

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**2.** *(a)* On the same Argand diagram sketch the locus of the points defined by the equations

(i) $z + z^* = 2$,

(ii) $\arg\left(\dfrac{z-2}{z+2}\right) = \dfrac{\pi}{4}$, where $\text{Im}(z) \geq 0$. **(6 marks)**

The region $R$ of the complex $z$-plane is defined by the inequalities

$$z + z^* \leq 2, \quad \arg\left(\frac{z-2}{z+2}\right) \geq \frac{\pi}{4} \quad \text{and} \quad \text{Im}(z) \geq 0.$$

*(b)* Shade the region $R$ on the Argand diagram. **(2 marks)**

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**3.** The points $A$, $B$ and $C$ with coordinates $(x_{-1}, y_{-1})$, $(x_0, y_0)$ and $(x_1, y_1)$ respectively lie on the curve $y = \text{f}(x)$ where $x_1 - x_0 = x_0 - x_{-1} = h$ and $y_n = \text{f}(x_n)$.

*(a)* By drawing a sketch, or otherwise, show that

$$\text{f}'(x_0) \approx \frac{\text{f}(x_0 + h) - \text{f}(x_0 - h)}{2h}.$$ **(3 marks)**

Given that

$$\text{f}'(x) = \sqrt{2x + \text{f}(x)}, \quad \text{f}(0) = 1 \text{ and } \text{f}(0.2) = 1.25,$$

*(b)* use two applications of the approximation in *(a)* with a step length of 0.2 to find an estimate for f(0.6).

**(5 marks)**

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**4.** The points $A$, $B$ and $C$ have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively such that

$$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = \mathbf{i} + q\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \mathbf{c} = 3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k},$$

where $q$ is a constant and $q \neq 2$.

*(a)* Find $\overrightarrow{AB} \times \overrightarrow{AC}$, giving your answer in terms of $q$. **(5 marks)**

*(b)* Hence show that the vector $\mathbf{n} = 4\mathbf{i} - \mathbf{k}$ is perpendicular to the plane $\Pi$ containing $A$, $B$ and $C$ for all real values of $q$. **(2 marks)**

*(c)* Find an equation of the plane $\Pi$, giving your answer in the form $\mathbf{r}.\mathbf{n} = p$. **(2 marks)**

Given that $q = -1$,

*(d)* find the volume of the tetrahedron $OABC$. **(3 marks)**

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**5.** *(a)* Use De Moivre's theorem to show that

$$\cos 5\theta \equiv \cos\theta(16\cos^4\theta - 20\cos^2\theta + 5). \quad \textbf{(6 marks)}$$

*(b)* By solving the equation $\cos 5\theta = 0$, deduce that

$$\cos^2\left(\frac{3\pi}{10}\right) = \frac{5-\sqrt{5}}{8}. \quad \textbf{(7 marks)}$$

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***Turn over***

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**6.** *(a)* Find the first three derivatives of $\ln\left(\frac{1+x}{1-2x}\right)$. **(6 marks)**

*(b)* Hence, or otherwise, find the expansion of $\ln\left(\frac{1+x}{1-2x}\right)$ in ascending powers of $x$ up to and including the term in $x^3$.

**(4 marks)**

*(c)* State the values of $x$ for which this expansion is valid. **(1 mark)**

*(d)* Use this expansion to find an approximate value for $\ln\frac{4}{3}$, giving your answer to 3 decimal places.

**(3 marks)**

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**7.** $\mathbf{A} = \begin{pmatrix} 2 & a & 2 \\ -1 & b & -2 \\ 0 & 0 & c \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 6 & 5 & 2 \\ -1 & 0 & -2 \\ 0 & 0 & 5 \end{pmatrix}$ and

$$(\mathbf{B} - 2\mathbf{I})\mathbf{A} = 3\mathbf{I} \quad \text{(i)}$$

where $a$, $b$ and $c$ are constants and $\mathbf{I}$ is the $3 \times 3$ identity matrix.

*(a)* Find the values of $a$, $b$ and $c$. **(6 marks)**

*(b)* Using equation (i), or otherwise, find $\mathbf{A}^{-1}$, showing your working clearly. **(2 marks)**

The transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is represented by the matrix $\mathbf{A}$.

*(c)* Find an equation satisfied by all the points which remain invariant under $T$.

**(4 marks)**

$T$ maps the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ onto the vector $\begin{pmatrix} 4 \\ -5 \\ 3 \end{pmatrix}$.

*(d)* Find the values of $p$, $q$ and $r$. **(3 marks)**

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**END**

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